

The image of addition

Aggregation



Augmentation



What is addition?

Year 1 Expectation

Linked resources and guidance

- [The image of addition](#)
- [Mental key skills and strategies](#)
- [Resources](#)
- [Key vocabulary](#)
- [Strengthening addition through reasoning](#)
- [Non-statutory notes and guidance](#)

Use objects and pictorial representations with the words orally and in written form, progress on to symbols when secure.

Use objects with number tracks and progress to fully marked number lines, then significant numbers marked if secure.

Statutory requirements

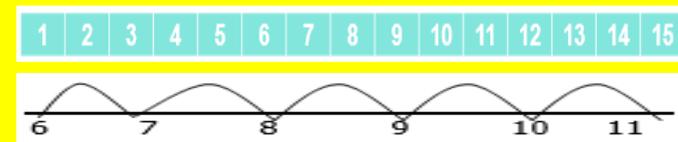
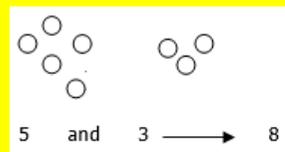
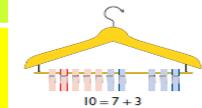
Pupils should be taught to:

- read, write and interpret mathematical statements involving addition (+), subtraction (−) and equals (=) signs
- represent and use number bonds and related subtraction facts within 20
- add and subtract one-digit and two-digit numbers to 20, including zero
- solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as $7 = \square - 9$.

Children need to be confident with:

- Counting, systematically lining objects up before point/ touch the objects to know the total
- having a true understanding of 'equals' linked to solving missing number problems e.g. $8 + 7 = \square$; $12 + \square = 18$; $16 + 0 = \square$; $17 = \square + 9$; $\square = 4 + 11$; $\square + \square + \square = 20$. What could the numbers be? What couldn't they be?
- number bonds to 20- build upon facts to 10 to approach facts to 20 with the same confidence
- interpreting statements with words as well as symbols 3 add 4 or 3 plus 4
- constructing statements from pictorial representations

Ensure children understand the concept of addition as:
combining groups of objects and **counting on**



Year 2 Expectation

Linked resources and guidance

- [The image of addition](#)
- [Mental key skills and strategies](#)
- [Resources](#)
- [Key vocabulary](#)
- [Strengthening addition through reasoning](#)
- [Non-statutory notes and guidance](#)

Strengthen understanding with concrete objects and pictorial representations.

Moving from fully marked number lines to partially marked, progressing to empty number lines where possible.

[Exemplification of strategy](#)

Statutory requirements

Pupils should be taught to:

- solve problems with addition and subtraction:
 - using concrete objects and pictorial representations, including those involving numbers, quantities and measures
 - applying their increasing knowledge of mental and written methods
- recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
- add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
 - a two-digit number and ones
 - a two-digit number and tens
 - two two-digit numbers
 - adding three one-digit numbers
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.

Children need to be confident with:

- recalling all number facts to 20 fluently
- the importance of zero
- counting on from given numbers in 10s (42, 52, 62,...) and linking this to calculation ($42 + 10 = ?$)
- reordering calculations to simplify e.g. $27 + 32$ would be better as $20 + 30$ then 7 and 2, or $32 + 20$ then + 7; knowing why the answer doesn't change whichever way the calculation is done
- solving missing number problems e.g. $47 + 9 = \square$; $\square + 30 = 45$; $36 = \square + 12$; $9 + \square + 5 = 16$ What could the numbers be? What couldn't they be?
- answering addition in one and **two step** problems

Year 2 continued

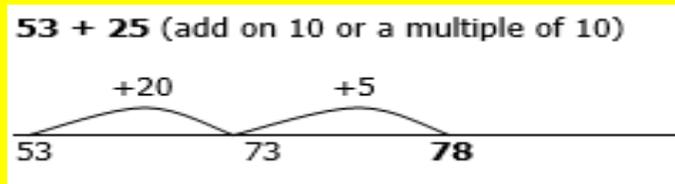
Ensure children understand the concept of addition as *combining groups* and *counting on*

$25 + 47 = ?$ $60 + 12$

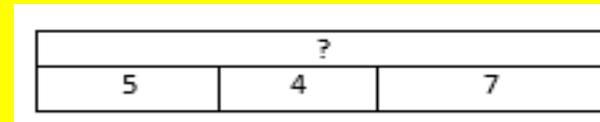
then exchanging:

$70 + 2 = 72$

progressing to larger numbers and developing efficiency using larger partitioned chunks on number lines



Use the bar model to build confidence with word problems
 Mubeena collects stickers. She has 5 animal stickers, 4 fruit stickers and 7 heart stickers, how many does she have in total?



Early development towards written methods in columns:

columns:

Use expanded place value, progressing to columns alongside concrete apparatus to support understanding

$$53 + 44 =$$

$$50 + 3 + 40 + 4 =$$

$$50 + 40 + 3 + 4 =$$

$$90 + 7 = 97$$

Alongside recording:

$$50 + 3$$

$$\underline{40 + 4}$$

$$\underline{90 + 7}$$

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Year 3 Expectation

Linked resources and guidance

- [The image of addition](#)
- [Mental key skills and strategies](#)
- [Resources](#)
- [Key vocabulary](#)
- [Strengthening addition through reasoning](#)
- [Non-statutory notes and guidance](#)

Strengthen understanding with concrete objects and pictorial representations.

Moving from partially marked number lines to empty number lines and partitioning for mental calculation.

[Exemplification of strategy](#)

Statutory requirements

Pupils should be taught to:

- add and subtract numbers mentally, including:
 - a three-digit number and ones
 - a three-digit number and tens
 - a three-digit number and hundreds
- add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

Children need to be confident with:

- place value up to 3 digits, including to 1 decimal place
- counting on in 10s (147, 157, 167,...) and linking this to calculation (147 + 10 = ? Extending to 147 + 30 = ?)
- linking counting on in 10s to counting on in 100s- explore for themselves and notice what happens to the digits e.g. 167, 267, 367, 467 which digit changes? Why? How is this different to 10s?
- reordering or rounding to simplify calculations, e.g. 132 + 9, could be 132 + 10 then subtract 1
- solving missing number problems e.g. $\square = 53 + 68$; $135 + 198 = \square$; $393 + 0 = \square$; $385 + \square = 185$; $567 + 300 = \square$; $670 = \square + 470$; $117 + \square + 64 = 199$ What could the numbers be? What couldn't they be?
- answering subtraction in one and two step problems

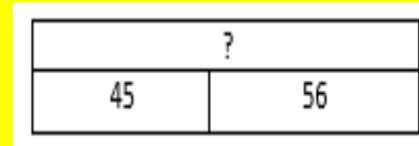
Ensure children use the number line or partitioning for mental addition, with larger numbers (up to 3 digits) and less jumps/ steps, and know when to use which depending upon the numbers involved

- Partition both numbers and recombine as in Y2
- Count on by partitioning the second number only

$$\begin{aligned}
 263 + 136 &= 263 + 100 + 30 + 6 \\
 &= 363 + 30 + 6 \\
 &= 393 + 6 \\
 &= 399
 \end{aligned}$$

Continued use of the bar model to aid confidence with problems

There are 45 apples and 56 pears, how many pieces of fruit altogether?



Column addition

Strengthening understanding with dienes before moving to place value counters (ideal for money)

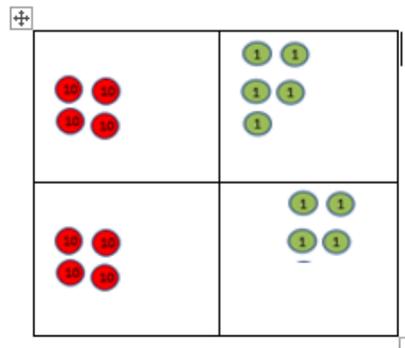
Without any exchanging first

Alongside expanded recording:

$$\begin{aligned}
 40 + 5 \\
 40 + 4 \\
 \hline
 80 + 9 = 89
 \end{aligned}$$

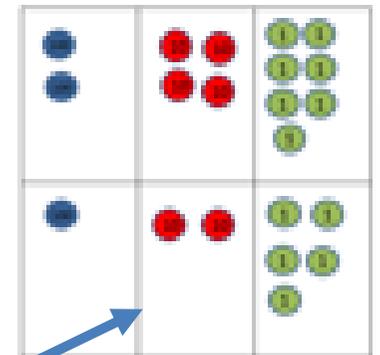
Leading to

45	45
<u>44</u>	<u>44</u>
80	to 9
<u> 9</u>	<u>80</u>
<u>89</u>	<u>89</u>



Leading to children understanding the exchange between tens and ones.

$$\begin{aligned}
 200 + 40 + 5 \\
 100 + 0 + 7 \\
 \hline
 300 + 40 + 12 = 352
 \end{aligned}$$



Progressing to exchanging and larger numbers

Progressing to the formal columnar algorithm initially introduced alongside the expanded method for some pupils.

Link to confidence with partitioning by starting with the most significant first-largest values

Grading of difficulty (+)

Year 3 continued

Year 4 Expectation

Linked resources and guidance

- [The image of addition](#)
- [Mental key skills and strategies](#)
- [Resources](#)
- [Key vocabulary](#)
- [Strengthening addition through reasoning](#)
- [Non-statutory notes and guidance](#)

Use pictorial representations and visualisation to strengthen understanding and to fully understand trickier calculations.

Use partitioning and empty number lines for mental calculation.

[Exemplification of strategy](#)

Statutory requirements

Pupils should be taught to:

- add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.

Children need to be confident with:

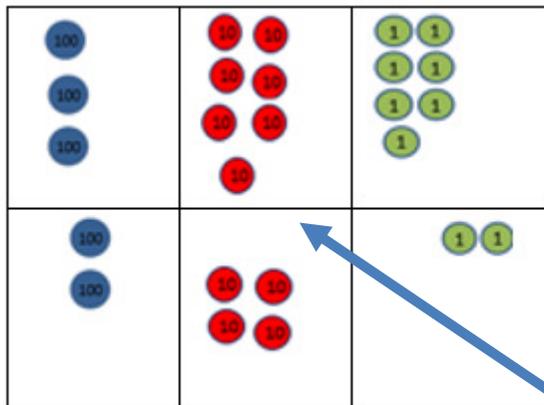
- place value up to 4 digits, including to 2 decimal places
- counting on in 1000s (1630, 2630, 3630,...) and linking this to calculation ($3630 + 1000 = ?$ Extending to $3630 + 3000 = ?$)
- linking counting on in 10s, 100s and 1/10s to counting on in 1000s- explore for themselves and notice what happens to the digits e.g. which digit changes? Why? How is this different to 10s and 100s?
- reordering or rounding to simplify calculations, e.g. $523 + 999$, could be $523 + 1000$ then subtract 1
- solving missing number problems e.g. $564 + \square = 760$; $\square + 163 = 467$; $250 + 199 + 99 = \square$; $800 + 75 + 125 = \square$; $3470 + 1000 = \square$; $\square + 3000 = 4900$; $26\square + \square 6\square = 500$
What could the numbers be? What couldn't they be?
- answering subtraction in multi- step problems

Ensure children use the number line or partitioning for mental addition (see Y2 and Y3), with more difficult numbers, including larger (up to 4 digits) and decimal numbers

Use of the bar model with problems if applicable

I have different lengths of rope: 5.4 m is red, 1.7 m is yellow, 2.5 m is blue. What length of rope do I have altogether?

?		
5.4 m	1.7 m	2.5 m



Columnar addition

Reinforce understanding of the expanded method from Y3, using place value counters to model

Without any exchanging, then with exchanging, with *decimals and up to 4 digits.*

377
242
 9
 110
500
 619

Leading to
 377
242
 619
 1

265.4 +
132.5
 0.9
 8.0
 90.0
300.0
397.9

It is a more efficient (streamlined) version of the expanded method, not a new method!

265.4 +
132.5
397.9

Progression to the formal columnar algorithm is expected by the end of the year.

Grading of difficulty (+)

Year 4 continued

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Year 5 Expectation

Linked resources and guidance

- [The image of addition](#)
- [Mental key skills and strategies](#)
- [Resources](#)
- [Key vocabulary](#)
- [Strengthening addition through reasoning](#)
- [Non-statutory notes and guidance](#)

Encourage pupils to visualise to fully understand trickier calculations, model understanding with pictorial representations.

Use empty number lines only if applicable.

[Exemplification of strategy](#)

Statutory requirements

Pupils should be taught to:

- add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
- add and subtract numbers mentally with increasingly large numbers
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

Children need to be confident with:

- counting on in 10s, 100s, 1000s, 10 000s, 1/10s and 1/100s and linking this to calculation ($2350 + 3000 = ?$ Knowing how to do this through counting on)
- linking counting on in 10s and 100s to counting on in 1000s and 10 000s- explore for themselves and notice what happens to the digits e.g. which digit changes? Why? How is this different to 10s/ 100s/1000s?
- rounding to simplify calculations and as a method to check accuracy of calculation
- Reordering to simplify calculations : $3.6 + 5.3 - 0.6$, $3.6 - 0.6 + 5.3 = 3 + 5.3$
- solving missing number problems e.g. $6.79 + \square = 14.8$; $1000000 + \square = 89000$; $36000 + 3\square000 + 800000 = 871000$; $15750 + 9750 = \square$; $11.94 = 11 + \square + 0.04$; What could the numbers be? What couldn't they be?
- answering addition in multi- step problems in all contexts

Continue to ensure children use the number line or partitioning for mental addition (see Y2 and Y3), with increasingly larger numbers, including up to 4 digits (and more than if applicable) and decimal numbers.

Continued use of the bar model to support problem solving if applicable

I win £200 in the lottery and decide to go shopping! I spent £79.99 in the first shop, £54.50 in the second and £24.50 in the third. How much did of my lottery winnings do I still have?

200			
?			?
£79.99	£54.50	£24.50	

Columnar addition

With secure understanding of the expanded method all pupils will be confident with the formal columnar algorithm.

Place value counters can continue to be used to support confident calculation with *decimals* and numbers with more than *4 digits* (see previous year exemplification).

$$\begin{array}{r}
 163.59 \\
 + \underline{55.68} \\
 \hline
 219.27 \\
 1
 \end{array}$$

$$\begin{array}{r}
 22762 + \\
 \underline{15203} \\
 \underline{37965}
 \end{array}$$

Year 5 continued

It is a more efficient (streamlined) version of the expanded method, not a new method!

Grading of difficulty (+)

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Year 6 Expectation

Linked resources and guidance

- [The image of addition](#)
- [Mental key skills and strategies](#)
- [Resources](#)
- [Key vocabulary](#)
- [Strengthening addition through reasoning](#)
- [Non-statutory notes and guidance](#)

The Y6 expectation is that children are fluent, and can calculate confidently using both mental and written strategies.

Less confident children should be encouraged to visualise or use resources.

[Exemplification of strategy](#)

Statutory requirements

Pupils should be taught to:

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- perform mental calculations, including with mixed operations and large numbers
- identify common factors, common multiples and prime numbers
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

135

Mathematics

Statutory requirements

- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

[Children need to be confident with:](#)

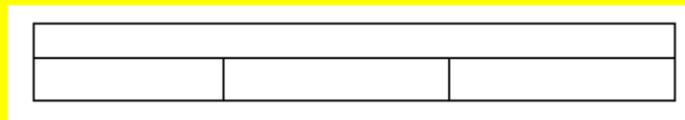
Y6 Children need to be confident with:

- place value with numbers greater than 4 digits, including to 3 decimal places
- counting on in 10s, 100s, 1000s, 10 000s, 1/10s, 1/100s and 1/1000s, and linking this to calculation ($15\ 950 + 30\ 000 = ?$ Or $6.152 + 1.09 = ?$ Knowing how to do this through counting on)
- linking counting on in 10s, 100s, 1000s, 10 000s, 1/10s, 1/100s and 1/1000s to counting on in and 100 000s and 1 000 000s- explore for themselves and notice what happens to the digits e.g. which digit changes? Why? How is this different to 10s/ 100s/1000s...?
- rounding to simplify calculations and as a method to check accuracy of calculation
- reordering to simplify calculations : $3.64 + 8.3 - 1.04$, $3.64 - 1.04 + 8.3 = 2.6 + 8.3$
- accurate conversion between all measures
- solving missing number problems e.g. $1\ 000\ 000 = 9\ 000\ 950 + \square + 125\ 000$; $\square = 143\ 500 + 14\ 650 + 75\ 950 + \square$; $5.12 + 6.\square\square = 12.74$; $\square = 845.91 + 72.19$; $2.145 + \square = 7.50$ What could the numbers be? What couldn't they be?
- experimenting with the order of operations and investigating the effect of positioning the brackets in different places, e.g. $20 - (5.0 + 3.5)$; $(20 - 5.0) + 3.5$
- [BODMAS](#)
- answering subtraction in multi- step problems in all contexts, with different mixed units and mathematics in use e.g. in the measuring jug I have 1.6 litres of liquid, 25% of it is orange juice, 6/10 is water, 50% of what is left is lemon juice- how much lemon juice is used?

Continue to ensure children use the number line or partitioning for mental addition (see Y2 and Y3), with increasingly larger numbers, including up to 4 digits (and more than if applicable) and decimal numbers.

Continued use of the bar model to support problem solving if applicable

I cut 3 strips of ribbon from a 2m length, the first piece is 65.9 cm, the second is 756 mm, the third is 0.156 m. How much ribbon have is left?



Columnar addition

All pupils should work confidently with the formal columnar algorithm, progressing to larger numbers and numbers to 3 decimal places.

$$\begin{array}{r} 237099 + \\ \underline{151955} \\ 389054 \\ 111 \end{array}$$

$$\begin{array}{r} 37.060 + \\ \underline{51.741} \\ 88.801 \\ 1 \end{array}$$

To secure both conceptual understanding and procedural fluency teachers may choose to model the method using the expanded method, pictorial or concrete apparatus as with previous years.

Year 6 continued

Alternative efficient written methods could help develop conceptual understanding, examples are available from:

[National Curriculum Appendix 1](#)

Grading of difficulty (+)

Key Vocabulary

The words listed below should be seen as a starting point, the more we consider which vocabulary is 'key' to strengthening understanding and supporting independent application the wider the vocabulary list grows.

Our children should be confident with all these words listed as well as being supported to find other vocabulary related to the four operations by themselves.

End of KS1	End of Y4	End of KS2
add, plus, total, altogether, combine, sum, also, as well as, how many more, increase, partition, calculate, largest, smallest, operation, double, near double, half, extra, inverse, estimate, re-order, more, equal, balance, jumps, steps, forwards, backwards, mentally,	efficient, written method, commutative, associative and distributive laws, decimal and values, expanded method, exchanging, algorithm,	Brackets, order of operations, BODMAS (or BIDMAS), aggregation, augmentation

Grading of difficulty (+)

This is an analysis of the difficulty of the calculation, and is independent of the method used to complete the calculation. Simpler calculations should be done mentally.

1	No carrying	+	$\begin{array}{r} 23 \\ 42 \\ \hline \end{array}$	+	$\begin{array}{r} 315 \\ 624 \\ \hline \end{array}$
2	Extra digit in answer	+	$\begin{array}{r} 94 \\ 73 \\ \hline \end{array}$	+	$\begin{array}{r} 561 \\ 718 \\ \hline \end{array}$
3	Carrying ones to tens	+	$\begin{array}{r} 47 \\ 25 \\ \hline \end{array}$	+	$\begin{array}{r} 237 \\ 516 \\ \hline \end{array}$
4	Carrying tens to hundreds	+	$\begin{array}{r} 371 \\ 485 \\ \hline \end{array}$	+	$\begin{array}{r} 293 \\ 541 \\ \hline \end{array}$
5	Carrying ones to tens and tens to hundreds	+	$\begin{array}{r} 376 \\ 485 \\ \hline \end{array}$	+	$\begin{array}{r} 295 \\ 547 \\ \hline \end{array}$
6	More than two numbers to be added	+	$\begin{array}{r} 35 \\ 62 \\ 24 \\ \hline \end{array}$	+	$\begin{array}{r} 237 \\ 148 \\ 516 \\ \hline \end{array}$
7	Different numbers of digits	+	$\begin{array}{r} 24 \\ 375 \\ 48 \\ \hline \end{array}$		

Other resources

- Support with making connections with the wider curriculum – NCETM www.ncetm.org.uk
- Problems Linked to the Primary National Curriculum for:
 - [Mathematics in EYFS, Year 1 and Year 2](#)
 - [Mathematics in Years 3, 4, 5 and 6](#)
- [Laws of mathematics](#)
- [PNS model and image charts](#)
- Interactive teaching programmes (ITPS) www.taw.org.uk/lic/itp