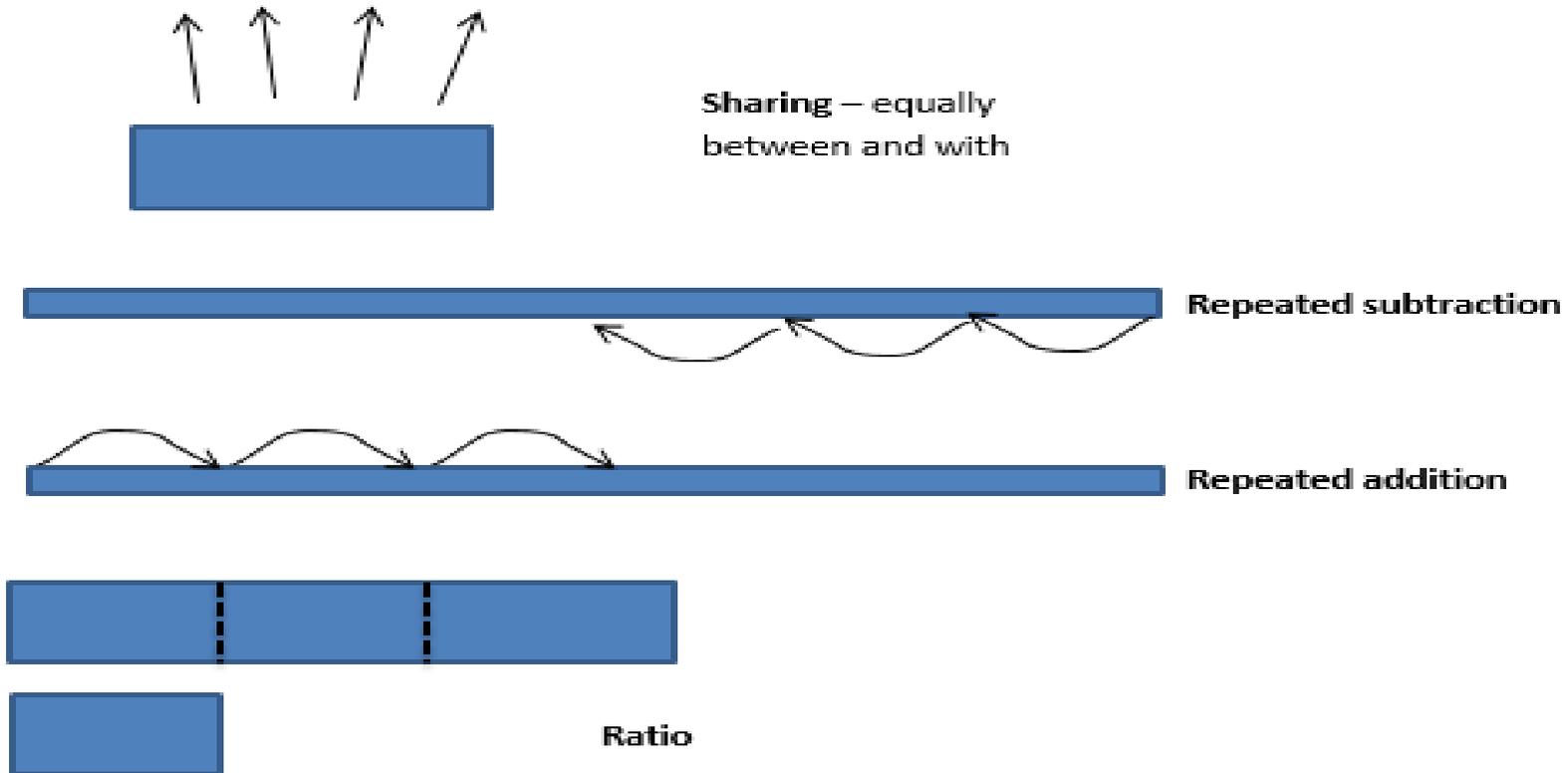


# The Image of Division



What is division?

# Year 1 Expectation

## Linked resources and guidance

- [The image of division](#)
- [Mental key skills and strategies](#)
- [Resources](#)
- [Key vocabulary](#)
- [Strengthening division through reasoning](#)
- [Non-statutory notes and guidance](#)

Use concrete objects and pictorial representations to share and group, progress to number tracks/lines if applicable

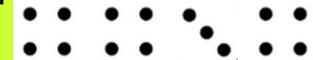
### Statutory requirements

Pupils should be taught to:

- solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Children need to be confident with:

- one to one correspondence
- counting backwards in 1s accurately and counting in groups of 2, 5 and 10, backwards as well as forwards
- doubling numbers up to 10, and halving numbers to 20
- making objects of equal group; recognising a group of objects and how it is different to another
- knowing the number of groups counted to support understanding of relationship between  $\times$  and  $\div$
- making connections, spotting patterns by themselves and recognising when something doesn't fit the pattern:
- having a true understanding of 'equals'
- simple missing number problems, e.g. The answer is  $\square$ , what have I doubled?;  $16 \text{ halved} = \square$

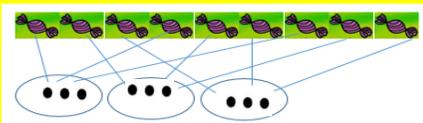


Ensure children understand the concept of division as:

**sharing between** (in to equal groups)

9 sweets are shared between 3 people.

How many do they have each?



and **sharing with- grouping** (including with remainders)

How many pairs of socks if you have

10 socks?



$$10 = 2 + 2 + 2 + 2 + 2$$

5 pairs

How many children can have 2 sweets out of this

bag of 12 sweets?



## Year 2 Expectation

### ***Linked resources and guidance***

- [The image of division](#)
- [Mental key skills and strategies](#)
- [Resources](#)
- [Key vocabulary](#)
- [Strengthening division through reasoning](#)
- [Non-statutory notes and guidance](#)

Strengthen understanding with concrete objects and pictorial representations.

Working with fully and partially marked number lines

[Exemplification of strategy](#)

### **Statutory requirements**

Pupils should be taught to:

- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication ( $\times$ ), division ( $\div$ ) and equals ( $=$ ) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Children need to be confident with:

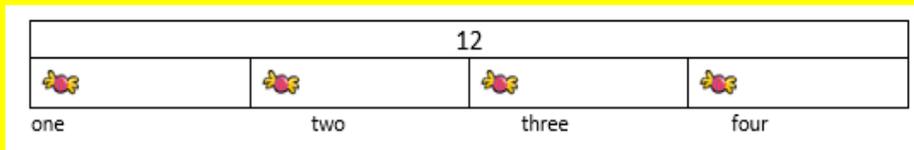
- counting back from given numbers in 2s, 5s and 10s and linking this to jumping back in repeated groups of, multiplication, and recognising groups in patterns: 5, 10, 15, 20... or 38, 36, 34, 32...
- true understanding of 'equals' and linking calculations with known key facts  $15 \div 5 = 5 \times 3$
- Partitioning numbers in different ways:  $24 = 20$  and 4 or 14 and 10...
- solving missing number problems e.g.  $\quad \div 2 = 4$ ,  $8 \div \quad = 4$ ,  $3 = 6 \div \quad$ ,  $16 \div \quad = 8$  What could the numbers be? What couldn't they be?
- recording statements using  $\div$  and  $=$
- answering division in one step problems

# Year 2 continued

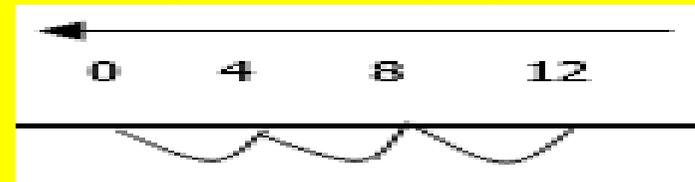
Continue to ensure that children understand the concept of division as **sharing between** (in to equal groups) and **sharing with- grouping** (including with remainders)

If I have 12 sweets and I share them equally between 4 children, how many does each child get?

Using the equipment in the bar model



If I have 15 sweets and I put them into bags of 4, how many bags will I have?



*progressing to larger numbers*

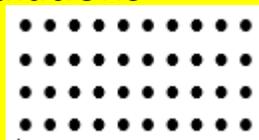
Linking to arrays- children drawing arrays for particular calculations

$$40 \div 4 = ?$$

How many groups of 4 in 34?

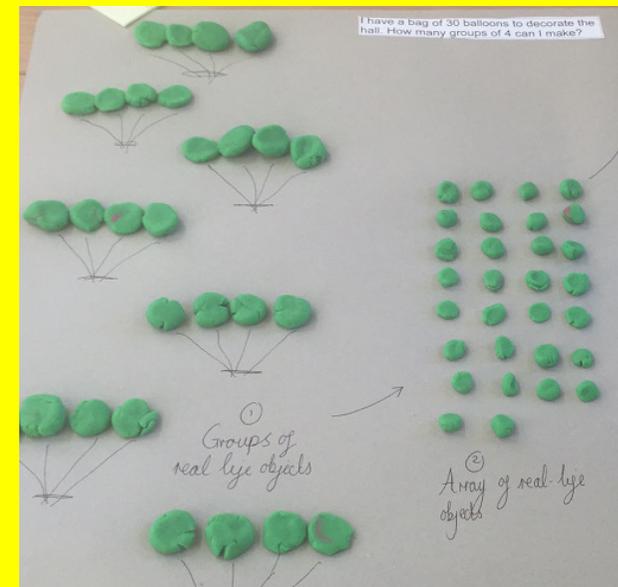


8 groups of 4, with 2 remaining



Use hands on equipment alongside the visual images

*develop efficiency using larger partitioned chunks for larger dividends*



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## Year 3 Expectation

### ***Linked resources and guidance***

- [The image of division](#)
- [Mental key skills and strategies](#)
- [Resources](#)
- [Key vocabulary](#)
- [Strengthening division through reasoning](#)
- [Non-statutory notes and guidance](#)

Develop understanding by modelling with concrete objects alongside pictorial representations.

Moving from partially marked number lines to empty number lines.

### **Exemplification of strategy**

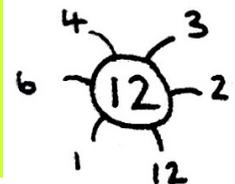
#### **Statutory requirements**

Pupils should be taught to:

- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which  $n$  objects are connected to  $m$  objects.

Children need to be confident with:

- place value and numbers to 100
- counting back from given numbers in 2s, 3s, 4, 5s, 8s and 10s, 50s, 100s and 1/10s and linking this to jumping back in repeated groups of, multiplication, and recognising groups in patterns: 8, 16, 24, 32... or 38, 34, 30, 26...
- linking calculations with known key facts  $32 \div 8 = 8 \times 4$
- partitioning numbers in different ways:  $54 = 50$  and  $4$  or  $34$  and  $20$ ...
- solving missing number problems e.g.  $\quad \div 2 = 24$ ,  $28 \div \quad = 14$ ,  $6 = 18 \div \quad$ ,  $45 \div \quad = 9$  What could the numbers be? What couldn't they be?
- doubling/ halving, knowing and applying understanding of factors and multiples
- using rounding and estimating to check
- answering division in one step problems



Continue to ensure that children understand the concept of division as **sharing between** (in to equal groups) and **sharing with- grouping** (including with remainders)

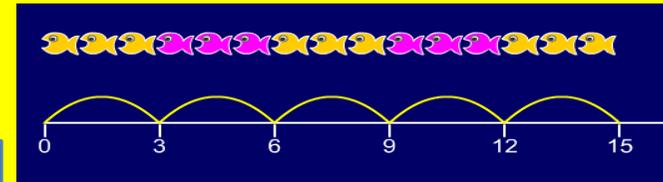
Continued use of the bar model to aid confidence with problems

Four children collected £16 in their book sale, how much did they earn each?

16			
Child 1	2	3	4

$$4 \overline{) 16}$$

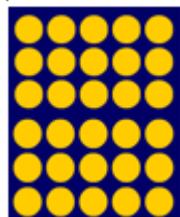
There are 15 fish in the pond, they swim along in groups of 3, how many groups are there?



**For sharing or grouping the approach to written methods is the same- it is the context that defines which one**

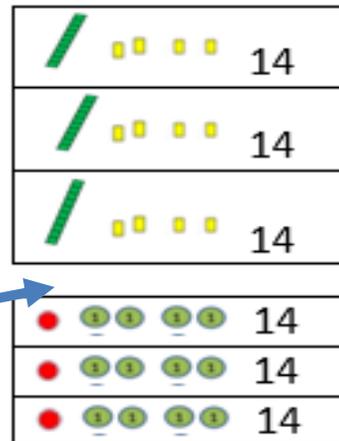
## Year 3 continued

Continuing work on arrays **interpreting** and drawing arrays for particular calculations  
 $32 \div 6 = ?$



Model the approach using hands-on equipment, progressing to dienes or place value counters to improve efficiency whilst moving to larger numbers

$$42 \div 3 = \square$$



Expanded alongside dienes and place value counters

$$\begin{array}{r} 10 + 4 \\ 3 \overline{) 40 + 2} \end{array} \quad \text{progressing to}$$

$$\begin{array}{r} 10 + 4 \\ 3 \overline{) 40 + 2} \end{array}$$

Progressing to expanded formal written methods only if applicable using concrete or visual support to aid understanding

**There is no curriculum expectation of standard algorithms for division yet!**

## Grading of difficulty (÷)

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## Year 4 Expectation

### **Linked resources and guidance**

- [The image of division](#)
- [Mental key skills and strategies](#)
- [Resources](#)
- [Key vocabulary](#)
- [Strengthening division through reasoning](#)
- [Non-statutory notes and guidance](#)

Strengthen understanding with pictorial representations, concrete equipment as needed to ensure children fully understand

Use empty number lines only if applicable.

### [Exemplification of strategy](#)

#### Statutory requirements

Pupils should be taught to:

- recall multiplication and division facts for multiplication tables up to  $12 \times 12$
- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as  $n$  objects are connected to  $m$  objects.

Children need to be confident with:

- place value and numbers to 3 digits
- counting back from given numbers in multiples up to  $12 \times 12$ , linking this to jumping back in repeated groups of, multiplication, and recognising groups in patterns: 7, 14, 21, 28... or 54, 48, 42, 36...
- linking calculations with known key facts  $48 \div 6 = 8 \times 6$
- partitioning numbers in different ways:  $72 = 70$  and 2 or 60 and 12, and using this simplify mental division calculations- which partitioning is more helpful? Why?
- solving missing number problems e.g.  $\div 12 = 6$ ,  $56 \div = 7$ ,  $6 = 54 \div$ ,  $81 \div = 9$  What could the numbers be? What couldn't they be?
- doubling/ halving, knowing and using factor pairs
- using rounding and estimating to check
- answering division in one step problems

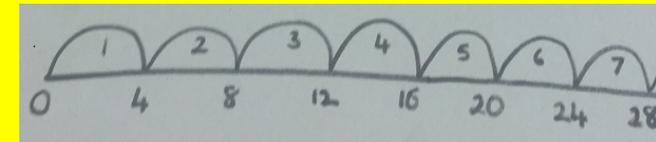
Continue to ensure that children understand the concept of division as **sharing between** (in to equal groups) and **sharing with- grouping** (including with remainders)

Use of the bar model with problems if applicable

84 marbles are shared between 7 children, how many did they each receive?

84	
70 x 10	14 = 7 x 2

Y4 need to raise £28 for some sports equipment for playtime, they earn £4 per week, how many weeks will it take them to earn it?



**For sharing or grouping the approach to written methods is the same- it is the context that defines which one**

As with Y3, reinforce understanding using visual support (concrete as needed).

Progressing to larger numbers

Approach the standard algorithm by expanding the digits alongside concrete equipment, before simplifying.

Although there is no curriculum expectation of standard algorithms for division yet children need to have the expanded building blocks for Y5

$$72 \div 3 = \square$$

		24
		24
		24
		24
		24
		24

Expanded alongside dienes and place value counters

$$\begin{array}{r} 20 + 4 \\ 3 \overline{) 60 + 2} \end{array} \quad \text{progressing to}$$

$$\begin{array}{r} 20 + 4 \\ 3 \overline{) 60 + 2} \end{array}$$

$$\begin{array}{r} 24 \\ 3 \overline{) 72} \end{array}$$

*It is a more efficient (streamlined) version of the expanded method, not a new method!*

**Grading of difficulty (÷)**

**Year 4 continued**

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**Year 5 Expectation**

***Linked resources and guidance***

- [The image of division](#)
- [Mental key skills and strategies](#)
- [Resources](#)
- [Key vocabulary](#)
- [Strengthening division through reasoning](#)
- [Non-statutory notes and guidance](#)

Encourage pupils to visualise to fully understand trickier calculations, model understanding with pictorial representations.

Use empty number lines only if applicable.

[Exemplification of strategy](#)

**Statutory requirements**

Pupils should be taught to:

- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- multiply and divide numbers mentally drawing upon known facts
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
- recognise and use square numbers and cube numbers, and the notation for squared ( $^2$ ) and cubed ( $^3$ )
- solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

[Children need to be confident with:](#)

# Y5 Children need to be confident with:

- place value and numbers to 4 digits
- counting backwards and forwards using a range of multiples (including decimals), and powers of 10, 100 and 1000
- linking calculations with known key facts  $48 \div 6 = 8 \times 6$ , so  $480 \div 60 = 80 \times 6$
- partitioning the dividend using multiples of the divisor to calculate mentally
- solving missing number problems e.g.  $\quad \div 20 = 140$ ,  $2.8 \div \quad = 1.4$ ,  $60 = 180 \div \quad$ ,  $450 \div \quad = 9$  What could the numbers be? What couldn't they be?
- know and apply [rules of divisibility](#)
- doubling/ halving, knowing and applying understanding of factors and multiples to calculate mentally
- common factors: 20- 1, 20, 2, 10, 10, 4 and 12- 1, 12, 2, 6, 3, 4. Highest Common Factor- 4, Lowest Common Factor- 1
- $\times$  and  $\div$  by 10, 100 and 1000 including within the context of measures
- make adequate estimates for calculations using rounding or knowledge of number facts
- answering division in multi- step problems in all contexts

Continue to ensure that children understand the concept of division as **sharing between** (in to equal groups) and **sharing with- grouping** (including with remainders)

Continued use of the bar model to support problem solving if applicable

There are 40 children in a class,  $\frac{2}{5}$  are girls, how many are boys?

40 children in total				
B	B	B	G	G
24 boys			16 girls	

Partitioning the dividend using multiplies of the divisor to calculate mentally

$$252 \div 12 =$$

$$240 \quad + \quad 12$$

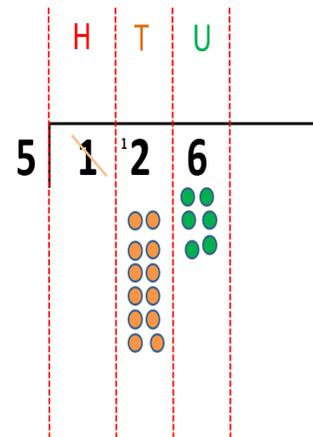
$$\downarrow \quad \quad \downarrow$$

$$12 \times 20 \quad + \quad 12 \times 1$$

$$252 \div 12 = 21$$

Understanding can continue to be supported using concrete equipment alongside visual images to confidently

Using place value counters to aid efficiency when working with larger numbers



$$137 \text{ r } 5$$

$$7 \overline{) 964}$$

**Year 5 continued**

With secure understanding of the expanded method all pupils should progress to the formal standard method **by the end of the year.**

**Grading of difficulty (÷)**

*It is a more efficient (streamlined) version of the expanded method, not a new method!*

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**Year 6 Expectation**

***Linked resources and guidance***

- [The image of division](#)
- [Mental key skills and strategies](#)
- [Resources](#)
- [Key vocabulary](#)
- [Strengthening division through reasoning](#)
- [Non-statutory notes and guidance](#)

The Y6 expectation is that children are fluent, and can calculate confidently using both mental and written strategies.

Less confident children should be encouraged to visualise or use resources.

[Exemplification of strategy](#)

**Statutory requirements**

Pupils should be taught to:

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
  - divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
  - divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
  - perform mental calculations, including with mixed operations and large numbers
  - identify common factors, common multiples and prime numbers
  - use their knowledge of the order of operations to carry out calculations involving the four operations
- 
- solve problems involving addition, subtraction, multiplication and division
  - use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

[Children need to be confident with:](#)

# Y6 Children need to be confident with:

- the place value of numbers greater than 4 digits, including decimals to 3 places
- counting backwards and forwards using a range of multiples (including decimals), and powers of 10, 100 and 1000
- linking calculations with known key facts  $72 \div 6 = 6 \times 12$ , so  $720 \div 60 = 60 \times 12$ , using them to scale up and down
- partitioning the dividend using multiples of the divisor to calculate mentally
- solving missing number problems e.g.  $\quad \div 20 = 1300$ ,  $280 \div \quad = 70$ ,  $600 = 18 \div \quad$ ,  $540 \div \quad = 0.9$  What could the numbers be? What couldn't they be?
- knowing and applying [rules of divisibility](#)
- knowing common factors, common multiples and prime numbers
- $\times$  and  $\div$  by 10, 100 and 1000
- accurate conversion between all measures
- rounding to simplify calculations and as a method to check accuracy of calculation
- making adequate estimates for calculations using rounding or knowledge of number facts
- [BODMAS](#)
- answering division in one multi- step problems in all contexts

Continue to develop confidence and understanding with mental division, with increasingly larger numbers, including up to 4 digits (and more than if applicable) and decimal numbers.

*Children could use the bar model, number line, informal jottings to support their mental computation*

$$1 \div 5 = 0.2$$

$$1 \div 8 = 0.125$$

$$1 \div 4 = 0.25$$

$$1 \div 0.25 = 4$$

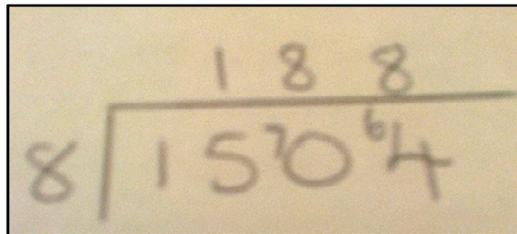
For every 2l water there is 60 ml of squash, if I use 270ml of squash how much water have I used?

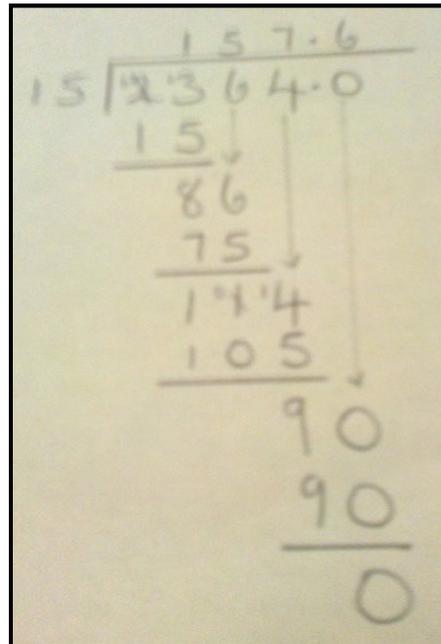
Water : Squash

2 l : 60 ml

? : 270 ml

Pupils should work confidently with using the formal written method for dividing numbers up to 4 digits by a two digit divisor


$$\begin{array}{r} 1888 \\ 8 \overline{) 15704} \end{array}$$


$$\begin{array}{r} 1576 \\ 15 \overline{) 23640} \\ \underline{15} \phantom{0} \\ 86 \phantom{0} \\ \underline{75} \phantom{0} \\ 114 \phantom{0} \\ \underline{105} \phantom{0} \\ 90 \phantom{0} \\ \underline{90} \\ 0 \end{array}$$

To secure both conceptual understanding and procedural fluency teachers may choose to model the method using the expanded method, pictorial or concrete apparatus as with previous years.

**Year 6 continued**

Alternative efficient written methods could help develop conceptual understanding, examples are available from:

[National Curriculum Appendix 1](#)

**Grading of difficulty (÷)**

# Key Vocabulary

The words listed below should be seen as a starting point, the more we consider which vocabulary is 'key' to strengthening understanding and supporting independent application the wider the vocabulary list grows.

Our children should be confident with all these words listed as well as being supported to find other vocabulary related to the four operations by themselves.

End of KS1	End of Y4	End of KS2
Divide, division, share, sharing with/ between, equal groups of, group, grouping, pairs, array, half, double, inverse, repeated subtraction, repeated addition, count back, count back from/ in, how many.., calculate, operation, left over/ extra, fractions- half, quarter, third, estimate, re-order, equal, balance, jumps, steps, forwards, backwards, mentally	Dividend, divisor, quotient, remainder, left over, <a href="#">rules of divisibility</a> , divisible by, factors, multiple, ratio, non unit fractions, efficient, exchanging, commutative, associative and distributive laws, written method, chunking, decimal and values, expanded method, algorithm	Brackets, order of operations, BODMAS (or BIDMAS), LCM, prime/ square and cube numbers, augmentation

# Grading of difficulty ( $\div$ )

This is an analysis of the difficulty of the calculation, and is independent of the method used to complete the calculation. Simpler calculations should be done mentally.

## One-digit divisor

<b>a</b>	No remainder, no exchange	$69 \div 3$	$264 \div 2$
<b>b</b>	Remainder, no exchange	$68 \div 3$	
<b>c</b>	No remainder, exchange	$45 \div 3$	
<b>d</b>	Remainder, exchange	$47 \div 3$	
<b>e</b>	Placing of the quotient	$287 \div 7$	
<b>f</b>	Noughts	$816 \div 7$	$5608 \div 8$

## Two-digit divisor

<b>g</b>	No remainder	$64 \div 32$	$93 \div 31$
<b>h</b>	Similar but remainder	$29 \div 13$	$97 \div 31$
<b>i</b>	Quotient not so apparent	$56 \div 22$	$92 \div 41$
<b>j</b>	Placing the quotient	$126 \div 21$	$224 \div 32$
<b>k</b>	No remainder	$483 \div 21$	$736 \div 32$
<b>l</b>	Remainder	$718 \div 33$	
<b>m</b>	Noughts	$683 \div 17$	

# Other resources

- Support with making connections with the wider curriculum – NCETM [www.ncetm.org.uk](http://www.ncetm.org.uk)
- Problems Linked to the Primary National Curriculum for:
  - [Mathematics in EYFS, Year 1 and Year 2](#)
  - [Mathematics in Years 3, 4, 5 and 6](#)
- [Laws of mathematics](#)
- [PNS model and image charts](#)
- Interactive teaching programmes (ITPS) [www.taw.org.uk/lic/itp](http://www.taw.org.uk/lic/itp)