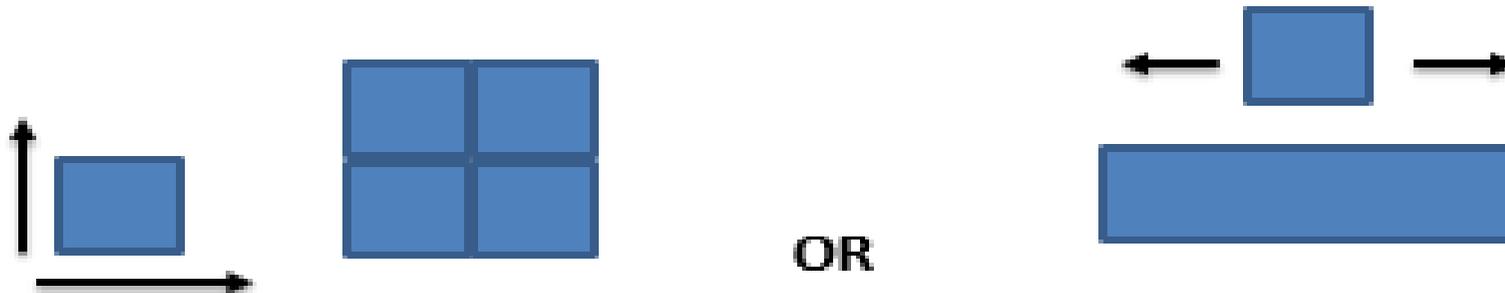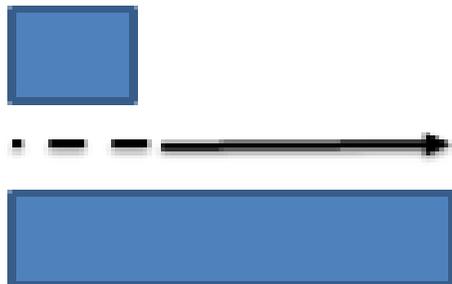# The Image of Multiplication

**Repeated aggregation**

**Scaling**

OR

OR

**What is multiplication?**

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# Year 1 Expectation

## Linked resources and guidance

- The image of multiplication
- Mental key skills and strategies
- Resources
- Key vocabulary
- Strengthening multiplication through reasoning
- Non-statutory notes and guidance

Use concrete objects and pictorial representations, use number tracks then lines to know how many groups there are

Children need to be confident with:

- one to one correspondence; counting forwards in 1s and in groups of 2, 5 and 10 accurately, forwards as well as backwards
- building and recognising **equal** groups, using **equal** groups to count, combining groups of the **same** size
- counting on the number of groups counted to support understanding of relationship between x and ÷
- doubling numbers up to 10, and halving numbers to 20
- making connections, spotting patterns by themselves and recognising when something doesn't fit the pattern:
- simple missing number problems, e.g. The answer is □, what have I doubled?; 4 doubled = □; true understanding of 'equals'
- all associated language- Ones, groups, lots of, repeated addition, groups of, lots of, times, columns, rows
- simple problems including with money and measures; recording what they have done practically with pictorial or symbol representation

Ensure children understand the concept of multiplication as **Repeated addition** (by combining groups)
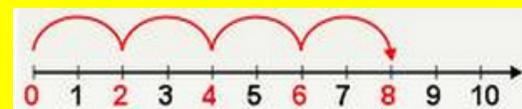
How many pairs of socks do we have in the washing basket?

$2 + 2 + 2 + 2 + 2 = 10$
5 groups of 2 socks
5 pairs
2 multiplied by 5
$2 \times 5 = 10$

If we all have two sweets each, how many do we need in total?

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# Year 2 Expectation

### Linked resources and guidance

- [The image of multiplication](#)
- [Mental key skills and strategies](#)
- [Resources](#)
- [Key vocabulary](#)
- [Strengthening multiplication through reasoning](#)
- [Non-statutory notes and guidance](#)

Strengthen understanding with concrete and real life objects as well as pictorial representations.

Use fully and partially marked number lines to model equal groups repeatedly to total.

[Exemplification of strategy](#)

## Statutory requirements

Pupils should be taught to:

- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers

- calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs

- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot

- solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Children need to be confident with:

- counting on from given numbers in 2's, 5's and 10's and linking this to jumping on in repeated groups of, and recognising groups in patterns: 5, 10, 15, 20… or 38, 36, 34…
- true understanding of 'equals' and linking calculations with known key facts $5 \times 4 = 20 \div 4$
- Partitioning numbers in different ways: 24= 20 and 4 or 14 and 10…
- solving missing number problems e.g. $8 = 2 \times$ ; $9 \times 2 =$ ; $= 5 \times 7$; $10 \times = 0$; $\times 6 = 12$ What could the numbers be? What couldn't they be?
- Constructing and interpreting arrays, and knowing that the column relates to the group size and the row relates to the number of repeated groups
- answering one step problems and recording statements using x and =

# Year 2 continued

Continue to ensure that children understand the concept of multiplication as **repeated addition** (by combining equal groups) and begin to understand **scaling**
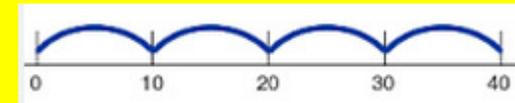
Begin to use of the bar model to aid confidence with problems

I get 10p for every night that I read at home, how much will I get if I read 4 times this week?

10 + 10 + 10 + 10 = 40p
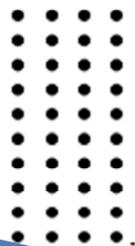4 groups of 10
4 10p coins
10 multiplied by 4
10p x 4 = 40p
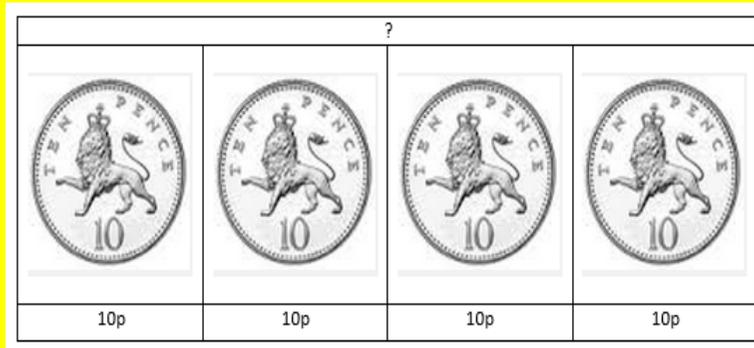
*...progressing to more jumps and larger numbers using number lines*

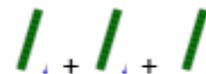Linking to arrays- children drawing arrays for particular calculations

10 x 4 = ?

Use hands on equipment alongside the visual images, progressing to dienes when confident

*Building blocks to written methods:*
Use partitioning to simplify multiplication of two digit numbers x by single digit   e.g. 13 x 3

10 + 10 + 10          and          3 + 3 + 3

and

10 x 3          and          3 x 3

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**Year 3 Expectation**

*Linked resources and guidance*
- The image of multiplication
- Mental key skills and strategies
- Resources
- Key vocabulary
- Strengthening multiplication through reasoning
- Non-statutory notes and guidance

Develop understanding by modelling with concrete objects alongside pictorial representations.

Moving from partially marked number lines to empty number lines.

Exemplification of strategy

**Statutory requirements**

Pupils should be taught to:
- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

Children need to be confident with:
- place value and numbers to 100
- counting on from given numbers in 2's, 3's, 4's, 5's, 8's and 10's, 50's, 100's and 1/10's, linking this to jumping on in repeated groups of, multiplication, and recognising groups in patterns: 8, 16, 24, 32… or 38, 34, 30, 26…
- linking calculations with known key facts $8 \times 6 = 48 \div 8$
- partitioning numbers in different ways: 64= 60 and 4 or 30 and 30 and 4…
- solving missing number problems e.g.    $x \; 4 = 32$, $8 \times \; = 64$, $27 = 9 \; x \;$, $5 \; x \; = 60$  What could the numbers be? What couldn't they be?
- doubling/ halving, knowing and applying understanding of factors and multiples
- using rounding and estimating to check
- answering division in one step problems

Continue to ensure that children understand the concept of multiplication as **repeated addition** (by combining equal groups) and **scaling**, strengthen their use of the number line for *larger numbers (See Y1 and 2)*
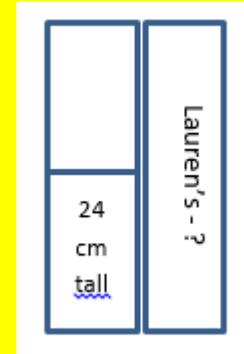
Continued use of the bar model to aid confidence with problems

Mrs Stokes bought a magazine each day during the week, it cost her £1.50 each day. How much did it cost for the 5 days in total?
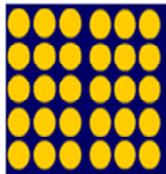
| How much in total? | | | | |
|---|---|---|---|---|
| £1.50 | £1.50 | £1.50 | £1.50 | £1.50 |

Lauren's sunflower has grown 2 times as tall as Josh's. Josh's sunflowers is 24 cm tall, how tall is Laurens?

24 cm tall | Lauren's - ?

## Year 3 continued

Continue working on **interpreting** and drawing arrays for particular calculations

5 x 6 = ?

*Progression to written methods:*
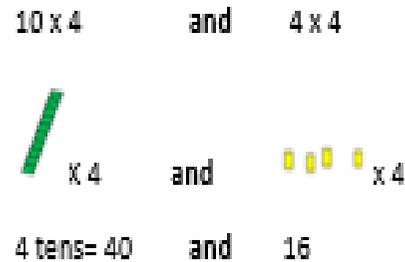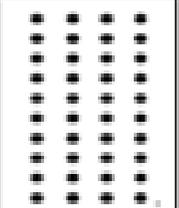
Model the approach using:

- objects progressing to dienes (or place value counters when secure)

- arrays in place value columns to improve efficiency whilst moving to larger numbers

10 x 4    and    4 x 4

X 4    and    x 4

4 tens= 40    and    16

Leading to arrays and grids:

| | 10 | 4 |
|---|---|---|
| 4 | • • • • <br> • • • • <br> • • • • <br> • • • • | • • • • <br> • • • • <br> • • • • <br> • • • • |
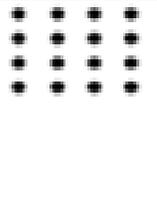
Progress to expanded written methods <u>only if applicable</u> using concrete or visual support to aid understanding

| Simplified to: | 10 | 4 |
|---|---|---|
| 4 | 40 | 16 |

**Grading of difficulty (x)**

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**Year 4 Expectation**

*Linked resources and guidance*
- The image of multiplication
- Mental key skills and strategies
- Resources
- Key vocabulary
- Strengthening multiplication through reasoning
- Non-statutory notes and guidance

Strengthen understanding with pictorial representations, dienes and place value counters to ensure children fully understand

Use empty number lines only if applicable.

Exemplification of strategy

Pupils should be taught to:
- recall multiplication and division facts for multiplication tables up to 12 × 12
- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

Children need to be confident with:
- place value and numbers to 3 digits
- counting on from given numbers in multiples of 25s, 1000s, 1/100s and up to 12 x 12, linking this to jumping on in repeated groups of, multiplication, and recognising groups in patterns: 7, 14, 21, 28… or 54, 48, 42, 36…
- linking calculations with known key facts 7 x 6 = 42 ÷ 6
- partitioning numbers in different ways: 156= 100, 50 and 6 or 150 and 6 or 70, 70 and 16- and using this simplify mental division calculations- which partitioning is more helpful? Why?
- solving missing number problems e.g.   x 12 = 60, 9 x   = 0, 108 = 12 x  , 81 =   x 9;  12 x   =   2 What could the numbers be? What couldn't they be?
- doubling/ halving, knowing and using factor pairs
- using rounding and estimating to check
- answering division in one step problems

Continue to ensure that children understand the concept of multiplication as **repeated addition** (by combining equal groups) and **scaling**, strengthen their use of the number line for *larger numbers (See Y1 and 2)*

As with Y3, reinforce understanding using visual and concrete equipment as needed allowing progression to larger numbers

Use of the bar model with problems if applicable

34 x 4

30 x 3        and        4 x 4

||||  X 3        and        x 4        x 3 and        x 4

9 tens = 90        and        16

I have £1.50, Liz has 3 times as much. How much does Liz have?

| £1.50 | £1.50 | £1.50 |

**Year 4 continued**

*Progression to written methods:*

*Using arrays, dienes, place value counters in grids:*
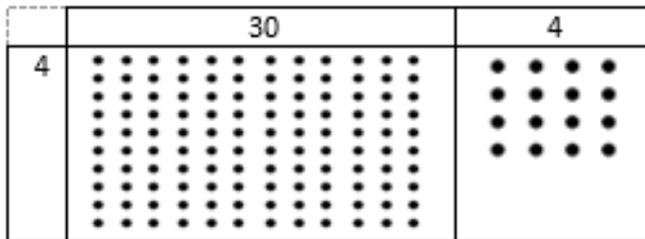
| | 30 | 4 |
|---|---|---|
| 4 | | |

| | 30 | 4 |
|---|---|---|
| 4 | | |

| Simplified to: | 30 | 4 |
|---|---|---|
| 4 | 120 | 16 |
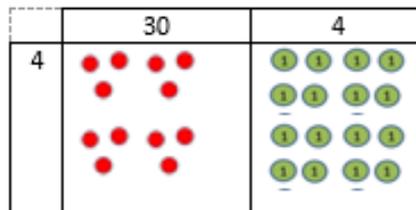
Leading to expanded written methods for **2 and 3 digit numbers multiplied by a 1 digit** number by the end of the year- use concrete equipment before simplifying

30 + 4
X          4
120 + 16

100 + 20 + 4
X              5
500 + 100 + 20

42
Leading to:  X   3
126

*It is a more efficient (streamlined) version of the expanded method, not a new method!*

**Grading of difficulty (x)**

**Year 5 Expectation**

*Linked resources and guidance*
- The image of multiplication
- Mental key skills and strategies
- Resources
- Key vocabulary
- Strengthening multiplication through reasoning
- Non-statutory notes and guidance

**Statutory requirements**

Pupils should be taught to:

- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- multiply and divide numbers mentally drawing upon known facts
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
- recognise and use square numbers and cube numbers, and the notation for squared ($^2$) and cubed ($^3$)
- solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

Encourage pupils to visualise to fully understand trickier calculations, model understanding with pictorial representations.

Use empty number lines only if applicable.

Children need to be confident with:

Exemplification of strategy

# Y5 Children need to be confident with:

- place value and numbers to 4 digits
- counting forwards using a range of multiples (including decimals), and powers of 10, 100 and 1000
- linking calculations with known key facts 8 x 9 = 72 ÷ 8, so 80 x 90 = 720 or 0.8 x 0.9 = 7.2
- partitioning and factorising to calculate mentally
- solving missing number problems e.g.    x 20 = 1.4, 25 x    = 750, 6 = 1.8  x    , 4500 x    = 900  What could the numbers be? What couldn't they be?
- doubling/ halving, knowing and applying understanding of factors and multiples to calculate mentally
- common factors: 20- 1, 20, 2, 5, 10, 4 and 12- 1, 12, 2, 6, 3, 4.  HCF- 4, LCF-1
- x and ÷ by 10, 100 and 1000 including within the context of measures
- make adequate estimates for calculations using rounding or knowledge of number facts
- answering multiplication in multi- step problems in all contexts

Continue to ensure that children understand the concept of multiplication as **repeated addition** (by combining equal groups) and **scaling**
*Children could use the bar model, number lines, informal jottings to support their mental computation for up to 4 digits and decimals*

Abdi's bike cost £127.50.  Ahmed buys a
bike that cost 3 times as much as
Abdi's, how much did Ahmed spend?

127.50

# Year 5 continued

Understanding can continue to be supporte using the grid or expand methods alongside concrete equipment (see Y4)

$$200 + 30 + 4$$
$$X \qquad 3$$
$$600 + 90 + 12$$

Leading to:
$$234$$
$$X \quad 3$$
$$702$$
$$1\ 1$$

## Columnar multiplication

With secure understanding of the expanded method all pupils should progress to the formal standard method **by the end of the year, with up to 4 digits multiplied by 1 and 2 digits**

$$100 + 50 + 6$$
$$X \qquad 10 + 3$$
$$300 + 150 + 18$$
$$1000 + 500 + 60 + \ 0$$
$$1000 + 800 + 210 + 18 \qquad = 2028$$

| Simplified or with resources as in Y4 | 60 | 3 |
|---|---|---|
| 30 | 1800 | 90 |
| 5 | 300 | 150 |

=    1890
=    315 +
      2205

Children must **understand** the importance of 0 as a place holder in these examples

Leading to:   156
$$X \quad 13$$
$$4\ _16\ _18$$
$$1560$$
$$2028$$
$$1\ \ 1$$

*It is a more efficient (streamlined) version of the expanded method, not a new method!*

## **Grading of difficulty (x)**

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**Year 6 Expectation**

*Linked resources and guidance*
- The image of multiplication
- Mental key skills and strategies
- Resources
- Key vocabulary
- Strengthening multiplication through reasoning
- Non-statutory notes and guidance

The Y6 expectation is that children are fluent, and can calculate confidently using both mental and written strategies.

Less confident children should be encouraged to visualise or use resources.

Exemplification of strategy

**Statutory requirements**

Pupils should be taught to:

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication

- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context

- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context

- perform mental calculations, including with mixed operations and large numbers

- identify common factors, common multiples and prime numbers

- use their knowledge of the order of operations to carry out calculations involving the four operations

- solve problems involving addition, subtraction, multiplication and division

- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

Children need to be confident with:

# Y6 Children need to be confident with:

- the place value of numbers greater than 4 digits, including decimals to 3 places
- counting forwards and forwards using a range of multiples (including decimals), and powers of 10, 100 and 1000
- linking calculations with known key facts 25 x 6 = 150 ÷ 12, so 25 x 0.6 = 15.0, using them to scale up and down
- solving missing number problems e.g.  x 20 = 14500, 2800 ÷  = 700, 60000 = 1200  x  , 54 x  = 0.9  What could the numbers be? What couldn't they be?
- knowing common factors, common multiples and prime numbers
- x and ÷ by 10, 100 and 1000
- accurate conversion between all measures
- partitioning and factorising
- rounding to simplify calculations and as a method to check accuracy of calculation
- making adequate estimates for calculations using rounding or knowledge of number facts
- BODMAS
- answering multiplication in one multi- step problems in all contexts

| £15995 | |
|---|---|
| £1595 | ? |

Columnar multiplication

All pupils should work confidently with the formal columnar algorithm, **with up to 4 digit numbers multiplied by 2 digits** including with decimals

$$200 + 40 + 7$$
$$\underline{X \qquad 20 + 5}$$
$$1000 + 200 + 35$$
$$\underline{4000 + 800 + 140 + \ 0}$$
$$\underline{4000 + 1800 + 540 + 35} \qquad = 6175$$

To secure both conceptual understanding and procedural fluency teachers may choose to model the method using the expanded method, pictorial or concrete apparatus as with previous years.

**Year 6 continued**

$$10 + 4 + 0.7$$
$$\underline{X \quad 20 + 5}$$
$$50 + 20 \ + \ 3.5$$
$$\underline{200 + \ 80 \ + 14.0}$$
$$\underline{250 + 100 + 17.5} \qquad = 367.5$$

Leading to:
$$2\ 4\ 7$$
$$\underline{X \quad 2\ 5}$$
$$1\ 2_2\ 3_3\ 5$$
$$\underline{4\ 9_1\ 4\ 0}$$
$$\underline{6\ 1\ 7\ 5}$$
$$_1$$

Alternative efficient written methods could help develop conceptual understanding, examples are available from:

*National Curriculum Appendix 1*

**Grading of difficulty (x)**

# Key Vocabulary

The words listed below should be seen as a starting point, the more we consider which vocabulary is 'key' to strengthening understanding and supporting independent application the wider the vocabulary list grows.

Our children should be confident with all these words listed as well as being supported to find other vocabulary related to the four operations by themselves.

| End of KS1 | End of Y4 | End of KS2 |
|---|---|---|
| Multiplication, equal groups of, multiplied by, lots of, pairs, array (rows and columns) half, double, inverse, repeated addition, repeated subtraction, count on/ up, how many.., calculate, operation, estimate, re-order, equal, balance, jumps, steps, forwards, backwards, double, near double, mentally | Product, multiplier, multiplicand, scaling, factors, multiple, ratio, non unit fractions, efficient, exchanging, commutative, associative and distributive laws, written method, decimal and values, expanded method, algorithm | Brackets, order of operations, BODMAS (or BIDMAS), aggregation, LCM, prime/ square and cube numbers, powers, |

# Grading of difficulty (x)

This is an analysis of the difficulty of the calculation, and is independent of the method used to complete the calculation. Simpler calculations should be done mentally.

| | | | | | |
|---|---|---|---|---|---|
| a | No 'carrying', answer less than 100. | × | 32<br>_3 | × | 44<br>_2 |
| b | Extra digit in answer. | × | 32<br>_4 | × | 51<br>_4 |
| c | 'Carrying' but keeping in same decade | × | 83<br>_4 | × | 34<br>_7 |
| d | 'Carrying' and going into next decade. | × | 78<br>_7 | × | 68<br>_8 |
| e | Noughts | × | 202<br>_4 | × | 430<br>_6 |
| f | Multiplying by multiples of 10 | × | 87<br>10 | × | 416<br>60 |
| g | Multiplying by two digits | × | 47<br>23 | × | 832<br>74 |
| h | Noughts | x | 802<br>74 | x | 820<br>74 |

# Other resources

- Support with making connections with the wider curriculum – NCETM [www.ncetm.org.uk](www.ncetm.org.uk)
- Problems Linked to the Primary National Curriculum for:
    - [Mathematics in EYFS, Year 1 and Year 2](Mathematics in EYFS, Year 1 and Year 2)
    - [Mathematics in Years 3, 4, 5 and 6](Mathematics in Years 3, 4, 5 and 6)
- [Laws of mathematics](Laws of mathematics)
- [PNS model and image charts](PNS model and image charts)
- Interactive teaching programmes (ITPS) [www.taw.org.uk/lic/itp](www.taw.org.uk/lic/itp)